Name:_____ Date:_____

Math 12 Honours Section 6.1 Imaginary and Complex Numbers

1. What is the difference between an "imaginary" number and a "complex" number? Explain:

2. Given a complex number: $z = 11 - 13i$, what is the value of $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$?

3. What happens whenever you multiply a complex number with it's conjugate?

4. Suppose $z + \overline{z} = 10$, what do you know about the $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$?

5. Suppose we are told that $z = \overline{z}$, what does this mean?

6. What does $|z|$ represent? What does it mean? Explain?

7. Given that $z_1 \times z_2 = 7 + 8i$, then what is the value of $\overline{z_1} \times \overline{z_2} = ?$

8. Given that $z_1 + z_2 = 55 - 45i$, then what is the value of $\overline{z_1} + \overline{z_2}$?

9. Solve for "x" and present your solution in the form of $a \pm bi$

a) $3x^2 - 2x + 7 = 0$	b) $\left(x^2 + 9\right)\left(x^2 + 100\right) = 0$	c) $7x^2 - 5x + 6 = 0$
d) $-2\left(x + 6\right)^2 + 1 = 65$	e) $4\left(x + 3\right)^2 + 25 = 0$	f) $x^4 + 16x^2 = 225$

g) $x^2-\left(\frac{2}{x}\right)^2=3$	h) $\frac{15}{x+3}-\frac{x}{x-3}=1$	i) $\frac{2}{x+5}-\frac{x}{x-5}=5$

10. Simplify or evaluate the following and express your answer in the form of $a\pm bi$:

a) $(3+2i)(1-3i)$	b) $\left(2-\sqrt{-4}\right)+\left(-3+\sqrt{-16}\right)$	c) $(-1+i)(i+1)+(3+i)(3-i)$
d) $\frac{1+i}{1-i}-\frac{1-i}{1+i}$	e) $\sqrt{\frac{-3}{2}}+\sqrt{\frac{-2}{3}}$	f) $\frac{1+2i}{3-4i}+\frac{2-i}{5i}$
g) $\frac{1+2i}{3+4i}+\frac{2i-5}{5i}$	h) $\left(\sqrt{9+40i}+\sqrt{9-40i}\right)^2$	i) $\frac{(2+i)^2}{2-i}+\frac{(2-i)^2}{2+i}$

11. Find the values of “a” and “b”:

a) $a+ib=\sqrt{153+104i}$	b) $a+ib=\sqrt{-16-30i}$	c) $a+ib=\sqrt{-15+112i}$

12. Given that “z” is a complex number in the form of $a\pm bi$, solve for “z”

a) $5z^2+4=0$	b) $z^2=5-12i$	c) $z^2=-3+4i$
d) $z^2+(i-5)z+12-5i=0$	e) $(5-2i)-(z+4i)=7-6i$	f) $z^3=8$

g) $z^2 - 15 + 8i = 0$	g) $z^2 - 16 - 16i\sqrt{3} = 0$	h) $z - \sqrt{-144} - \left(3\sqrt{-i} + 1\right)^2 = 7 - 6i$

13. Evaluate $i^{2021} \times i^{2020} \times i^{2019} \times i^{2018}$

14. Find the value of $\left(-i\right)^{4n-1}$ when "n" is a negative odd integer.

15. If "z" is a complex number and $\bar{z}$ is its conjugate, then determine the complex numbers which satisfy the equation: $5z^2 - 4z\left(\bar{z}\right) = \left(1 - 3i\right)z$

16. Given that $f\left(x\right) = \left(-2 + i\right)x^2 - \left(3 + i\right)x + 4 - 5i$, find the value for each of the following:

i) $f\left(i\right)$ ii) $f\left(1 + i\right)$ iii) $f\left(3 - i\right)$

17. If “z” is a complex number and $\bar{z}$ is its conjugate, then determine the value of : $z^5 - \left(\bar{z}\right)^5$

18. Find the sum of the following: $1 + 2i + 3i^2 + 4i^3 + ... + 1000i^{999} + 1001i^{1000}$

19. There is a complex number “z” with imaginary part 164 and a positive integer “n” such that: $\frac{z}{z+n} = 4i$.
    Find the value of “n”. AIME I 2009

20. Find “c” if “a”, “b”, and “c” are positive integers that satisfy the following equation: $c = \left(a + ib\right)^3 - 107i$
    1985 AIME

21. Given the following equation, find the value of “k” if “k” and “m” are integers:

$$\left[2-\left(-2+i\sqrt{3}\right)-\left(-2-i\sqrt{3}\right)\right]\left[2+\left(-2+i\sqrt{3}\right)^2+\left(-2-i\sqrt{3}\right)^2\right]\left[2-\left(-2+i\sqrt{3}\right)^4-\left(-2-i\sqrt{3}\right)^4\right]=2^k3^m$$

21. Find the number of ordered pairs of real number (a,b) such that $\left(a+ib\right)^{2002}=a-bi$ (AMC 12)

a) 1001 b) 1002 c) 2001 d) 2002 e) 2004